1. Introduction

Tatamibari is a puzzle popularized by Nikoli, a puzzle magazine from Japan, in the 1990s. Tatamibari puzzle has been proven as NP-complete in [1]. Tatamibari puzzle is a puzzle that consists of $m \times n$ grid of cells and each cell either has one of the hints or is empty. The hints consists of $|$, $-$, or $+$. The objective of the Tatamibari puzzle is to fill the empty cells with hints such that:

1. Every cell has exactly one identity.
2. The shape of every partition (i.e., the collection of cells with the same identity) must be rectangular.
3. The shape of every partition depends on the hint described in any cell within that partition, namely:
   (a) If a cell contains '$+$', the it shape must be a square.
   (b) If a cell contains '$-$', the it shape must be a rectangle and its width must be greater that its height.
   (c) If a cell contains $|$, the it shape must be a rectangle and its width must be smaller that its height.
4. There is no four adjacent cells, namely (r,c), $(r,c+1)$, $(r+1,c)$, and $(r+1,c+1)$ (for some $1 \leq r \leq m$ and $1 \leq c \leq n$ where $m \times n$ is the size of the Tatamibari board) with distinct identities (i.e, the identity of (r,c), $(r,c+1)$, $(r+1,c)$, and $(r+1,c+1)$ cannot all be different).

Puzzle is a form of entertainment that gives a feeling of satisfaction to the player upon completing it [12]. Puzzle also contains mathematical and computational aspects that have connections with combinatorial and computational problems. On the topic of puzzle complexity, numerous systematic research have been conducted [4, 12, 9]. Taken from [12], there has been many puzzle that have been proven to be NP-complete, such as : Blocks World, Clickomania, Corral Puzzle, Cross Sum, Cryptarithms, Instant Insanity, KPlumber, Lemmings, Light Up, Mastermind, Minesweeper, n-Puzzle, Nurikabe, Pearl Puzzle, Peg Solitaire, Reflections, Rush Hour, Shanghai, Slither Link, Sokoban, Solitaire, Spiral Galaxies, Sudoku, Tetris. Additionally, Tatamibari [1] and Yin-Yang [6] is also an NP-complete puzzle.

Tatamibari is an NP-complete puzzle means it must have a Tatamibari solution verifier that can be executed in polynomial time. Moreover, a Tatamibari puzzle solver that can be executed in exponential time should also exist. However, the algorithmic investigation of the Tatamibari puzzle is relatively new and limited; to our knowledge, a formal investigation of the Tatamibari puzzle has never been discussed. There are several approach to solve an NP-complete puzzle, such as using integer programming model [7] or the SAT-solver [13, 18, 15, 14, 16]. In this paper, we discuss an explicit yet elementary techniques for solving arbitrary Tatamibari puzzles, the exhaustive search approaches and SAT-solver. We show that we can find all solutions of arbitrary Tatamibari puzzles in exponential time in terms of the size of the puzzle and the number of hints.

The rest of the paper is organized as follows. We discuss some theoretical aspects of the Tatamibari puzzles in Section 2.. In Section 3. we discuss an $O(hmn)$ time algorithm for verifying whether an arbitrary $m \times n$ Tatamibari configuration is also a solution. We discuss the exhaustive search algorithm in Section 4. and show that we can solve an arbitrary $m \times n$ Tatamibari puzzles with h hints in $O(\max\{m^2n^2, h^{mn-h} \cdot hmn\})$ time. We also discuss the SAT solver approach in Section 5. and derives the asymptotic running time for solving Tatamibari puzzles using SAT solver. Section 6. discusses computational experiments of our algorithms. Finally, this paper is summarized and concluded in Section 7..