1. Introduction

Yin-Yang (also known as Shiromaru-Kuromaru) is a pencil-and-paper puzzle first published in 1994 by the Japanese magazine Puzzler that has recently been proven to be NP-complete in [12]. In general, a Yin-Yang puzzle consists of $m \times n$ grid of cells, and every cell either has a black circle, a white circle, or is empty. The objective of this puzzle is to fill every empty cell with either a black or a white circle such that: 1) for each color (black and white), the cells containing circles of the same color form a single connected group of cells, where connectivity is based on four-way orthogonal adjacency; 2) there is no 2×2 grid of cells containing the same color.

Puzzles, particularly those based on pencil-and-paper, are primarily intended as recreational tools [29]. Nevertheless, the mathematical and computational aspects of puzzles have undergone significant investigations due to their connections to important combinatorial and computational problems. Several systematic studies have been carried out on the topic of the complexity of puzzles [10, 25, 16]. Moreover, many pencil-and-paper based puzzles have been proven to be NP-complete, such as (in chronological order): Sudoku (2003) [39], Nurikabe (2004) [17], Hiroimono (2007) [5], Heyawake (2007) [18], Hashiwokakero (2009) [6], Kurodoko (2012) [26], Yajilin and Country Road (2012) [20], Shikaku and Ripple Effect (2013) [36], Yosenabe (2014) [21], Shakashaka (2014) [13], Fillmat (2015) [37], Usowan (2018) [22], Sto-Stone (2018) [4], Dosun-Fuwari (2018), [23], Tatamibari (2020) [3], Kurotto and Juosan (2020) [24], and Yin-Yang (2021) [12].

The NP-completeness of Yin-Yang puzzles means that there should exist a Yin-Yang solution verifier that can be executed in polynomial time. Moreover, there also should exist an exponential time algorithm for solving arbitrary Yin-Yang puzzles. Nevertheless, since algorithmic investigation relating to the Yin-Yang puzzle is relatively new and limited, to our knowledge, a formal investigation of the Yin-Yang solver has never been discussed rigorously. There are numerous approaches for solving NP-complete puzzles, such as using the integer programming model [13] or the SAT-solver technique [27, 38, 31, 30, 35]. Nevertheless, in this paper, we discuss two explicit yet elementary techniques for solving arbitrary Yin-Yang puzzles, namely the exhaustive search and prune-and-search approaches. In addition, we discuss a Yin-Yang solver using SAT solver. We show that we can find all solutions of arbitrary Yin-Yang instances in exponential time in terms of the size of the puzzle and the number of hints.

The rest of the paper is organized as follows. We discuss some theoretical aspects of the Yin-Yang puzzles in Section 2.. Here, we also derive an additional rule regarding the non-existence of a 2×2 alternating pattern. In Section 3. we discuss an $O(mn)$ time algorithm for verifying whether an arbitrary $m \times n$ Yin-Yang configuration is also a solution. We discuss our main algorithms in Section 4. and show that we can solve an arbitrary $m \times n$ Yin-Yang puzzles with h hints in $O(\max\{mn, 2^{mn-h}\})$ time. We also discuss SAT solver implementation in Section 5. along with its Python implementation. Section 6. discusses computational experiments of our algorithms. Here, we also discuss some combinatorial results based on mathematical analysis and experiments. Finally, this paper is summarized and concluded in Section 7..