

Classification of Crackle Sound on Lung Using Discrete Wavelet Transform(DWT) and Restricted Boltzmann Machine(RBM)

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Abstract

A crackle sound on lung happen because there's a anomaly on respiratory tract. Crackle sound like a rattling or clicking that happen to be heard when inhaling or exhaling or both phase of breathing. Auscultate is method that use to evaluate abnormalities inside respiratory tract but this method are still a subjective method. There's a lot of research that using this problem as its goal with using a different kind of method like using features extraction method, one of those is Discrete Wavelet Transform(DWT). Using Wavelet Transform help to separate the crackle feature from lung sound and using a classification to classify it characteristic. the method is easy to use on wave form of data and it used on this final project. The extracted features classify and tested using Restricted Boltzmann Machine(RBM) which resulting 69% as highest accuracy result.

Keywords: lung sound, features extraction, RBM, DWT, crackle.

1. Introduction Background

Crackle sound in lung happen because there is some anomaly on respiratory tract. The crackle sounds like a rattling or clicking that occur when inhaling, exhaling, or both. Duration of crackle sound is 20 ms more or less and located at 100 until 2 kHz on frequency range[1]. The number of crackle can occur is related to timing, duration, and how bad the disease were[2]. The anomaly on respiratory tract will change the sound of lung itself[3]. Auscultate method is can be use to evaluate its abnormalities. This is a check up that fast, efficient, non-invasive, and cheap[4]. Even if the auscultate method using stethoscope usually performed to identify crackle sound on lung, but the diagnose is still subjective which is why a lot of doctor are counting on other method to evaluate lung condition.

A significant effort has done on applying signal processing and made up intelligent in order to classify crackle sound in lung[1]. A lot of method that is successfully implemented on classifying crackle sound on lung like using Tsallis Entropy as a features extraction and Multilayer Perceptron with the result of it reach 95.35%[3]. There are also a research that use Discrete Wavelet Transform(DWT) and Artificial Neural Network(ANN) classifier with 265 segment of time, resulting 100% of accuracy on training set and 94.02% on validation set[5].

Because of the result on this [5] research giving out an outstanding accuracy, therefore for features extraction method will be using Discrete Wavelet Transform(DWT). DWT works on signal by divide the signal into half of its original size, and then that side will be processed by using low-pass filter and high-pass filter. Those process will be repeated depend what wanted. After reaching level we wanted, student will use the chosen level for its features to be extracted. The author using this method as features extraction method because the accuracy that get from this will not bad and also DWT is often use on signal processing, easy to implement and easy to understand.

There are a lot of method that could provide a good accuracy and a lot of those method are not often use on classifying crackle sound or entire lung sound itself and one of the is Restricted Boltzmann Machine(RBM). RBM is a generative model that study the distribution probabilities through input. It also a probabilistic graphic model that can be interpretate as stochastic neural network[6]. In RBM, there is one hidden layer and one visible layer[7]. Each unit in hidden layer aren't connected to other hidden layer unit and same with visible layer unit. The reason using this method is to know if RBM is well enough to classify crackle extracted features. That why the author will be using Discrete Wavelet Transform(DWT) and Restricted Boltzmann Machine(RBM).

Problem statement

Base on background that been explain above, therefore problem from this final project would be:

1. How to built a crackle sound on lung classification system by using Discrete Wavelet Transform(DWT) and Restricted Boltzmann Machine(RBM)
2. How the performance of the classification system after being train with train data with different approach of training data. How would the accuracy become

There is a limitation regarding the problem statement in this final project which is:

1. Lung sound that will be use only 274 total because was what being provided
2. txt file from lung sound data will be convert to csv file because to ease the process of extracting data inside file
3. classification will do all class that declare inside file but but the main focus was only normal and crackle sound.

Purpose

The goal of this final project is:

1. to built a crackle sound on lung classification system using Discrete Wavelet Transform (DWT) and Restricted Boltzmann Machine (RBM)
2. to find out how well the classification using Restricted Boltzmann Machine (RBM) do on classify crackle sound on lung

2. Literature Review

An embedded classifier of lung sounds based on the wavelet packet transform and ANN with signal processing topic and using Wavelet Packet Transform and Artificial Neural Network receive 99.26% success rate on ANN classification system[1].

A lung sound classification system base on the rational dilation wavelet transform with signal processing topic and using Rational dilation Wavelet Transform, Support Vector Machine and kNearest Neighbors resulting k-NN with crackle detection rate achieved by the energy subnet feature with 95.00%, the highest wheeze detection rate is achieved using subnet entropy with 98.00% and the highest detection rate of normal signal is achieved by standard deviation of subnet with 95.00% is 95%[2].

Pulmonary crackle feature extraction using tsallis entropy for automatic lung sound classification with signal processing topic using Tsallis Entropy resulting Tsallis entropy produces high accuracy reaching 95.35%[3].

Pulmonary crackle characterization: approaches in the use of discrete wavelet transform regarding border effect, mother wavelet selection, and subband reduction with border extension topic using Discrete Wavelet Transform resulting that extension modes considered during DWT affected crackle characterization, whereas SP1 and ASYMW modes should not be used DWT[4].

Neural classification of lung sounds using wavelet coefficients with signal processing topic using Wavelet coefficients an Artificial Neural Network resulting The 1940-6 ANN architecture was found as the optimum model for classification using statistical features of Wavelet coefficients[5].

Training Restricted Boltzmann Machines with classification topic using Restricted Boltzmann Machine give an explanation of RBM from the perspective of probabilistic graphical models[6].

Review of Boltzmann machines and simulated annealing with classification topic using Restricted Boltzmann Machine explaining the basis of Boltzmann's classification method and its usage[7].

3. Material and Method

In general, the design and implementation stages of the final task are explained in the whole system flow chart, as follows:

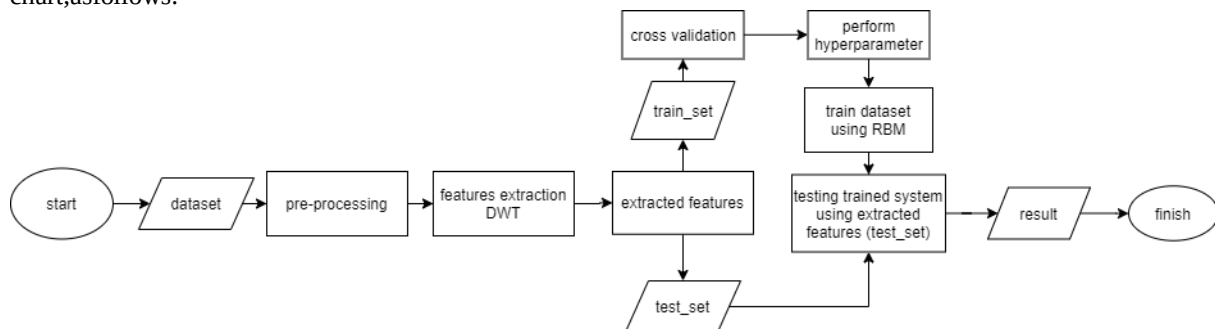


Figure 1. system flow

First dataset will be pre-processed by implementing couple step which is splitting the signal into couple segment, frequency resampling each of those segment, then applying moving average filter. With data that been pre-processed, data will be inputted into Discrete Wavelet Transform(DWT). In DWT, the lung sound data will be decompose to 7 level subbands, number of subbands is depends on what frequency sampling that applied to the lung sound and location of crackle sound in signal. From decomposing, each subbands by using the statistical features extraction that already explain.

After extraction, the data will be store first in one csv and also will load the data class from the text file. Before being use inside RBM, first data will be loaded and separate data from its class for both train data and test data. After all dataset are loaded, data will be rescaling to 0-1 range to ease the time of training and for train data will be inputted to cross validation so that the accuracy from CV will be use as comparison to the system accuracy. The before training, training set will be use for hyperparameter tuning to find out which of parameter is best fit for the training. Training the dataset using RBM with defining its learning rate, how many hidden unit will be use, use gibbs sampling to train the train data. Train will be conducted using some iteration that defined. After done training the data, test data will be inputted to trained system and the output will be classify by Softmax the system will outputting the accuracy and number of data that successfully classify.

Data

The data that provide from lecturer is 274 data total including txt file that consist of inspiration time, expiration time, and class or the diagnosis of the lung sound. Lung sound data is in format .wav will be split into couple segment base on their inspiration time and expiration time for each cycle of breathing. Subject of data consist of normal, crackle, and wheeze. Class subject will be convert to number to ease system to classify.

Pre-processing

The pre-processing will conduct several step to make sure that the data signal is able to be use as an input whether to the feature extraction method or the classification itseft. the step that will attempt is:

- Split into couple segment
Split the lung sound into couple segment base on the inspiration and expiration from txt file
- Resample frequency
Resample the frequency so that signal In the frequency range that able to ease the features extraction method to process the signal.
- Normalization
Normalization is to reduce sharp and sudden changes of the signal
- Perform a moving average filter
Applying moving average filter to reduce noise and smoothing the signal

Crackle Sound

Crackle can occur due to turbulence in the flow of air flowing in the airways due to a some fluid that blocks the flow of air in the airways. Crackle has a characteristic form of sharp, sudden deflection which is often followed by waves[8]. Duration on one cycle of respiration is 20 ms more or less and can be found on 100 until 2 kHz frequency range[1]. Frequency and duration from crackle sound is determine by the obstacles location, in and out wind acceleration, and respiratory tract diameter.

Crackle can be heard in inspiration only, expiration only, or can be heard from both inhale and exhale process. Crackle on inspiration occurs because when breathe in, air flows through channels that are blocked by fluid and the same thing happens during expiration. The barrier fluid in the respiratory tract can change its position and diameter when the patient coughs.

Discrete Wavelet Transform(DWT)

In general, discrete wavelet transform (DWT) can be stated by:

$$X(a, b) = \frac{1}{\sqrt{b}} \int_{-\infty}^{\infty} x(t) \Psi\left(\frac{t-a}{b}\right) dt, \quad (1)$$

Where a is time shift and b is a scale(often call of modulation width), $x(t)$ is input signal, $\Psi(t)$ called wavelet function or mother wavelet[9]. Wavelet is form using finite impulse respond of low-pass and high-pass[10]. In mathematic, the output of the filter can be declare by:

$$Y_{HP}[t] = \sum u[a].g[2t - a] \quad (2)$$

$$Y_{LP}[t] = \sum u[a].h[2t - a]$$

Where u is represent the original signal and g, h variables represent Finite Impulse Response (FIR) of low-pass and high-pass filter. It can be seen in picture 1[11] that the signal will be split evenly from its normal frequency range. The signal will be downsampled by 2 and number of sample will become half of it original size[11]. In DWT, only low-pass output that get to approximate and will be downsampled again and so on. DWT allow to decomposition the signal to two element which is approximation (high scale with low frequency component) and detail element(low scale with high frequency component)[10].

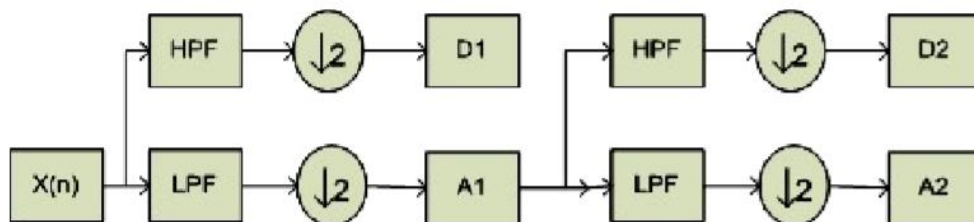


Figure 2. wavelet decomposition levels[11]

In DWT, 7 level that performed by Kandaswamy on [5] research to decompose lung sound using 11025hz frequency sampling. Same scenario perform by Hashemi in [12] reseach but different with Kandaswamy, the frequency sampling that used is 8000hz. DWT is depended to its frequency sampling[11], and range of each subband from [5] and [12] reseach can be seen from *table.1* below:

Table 1. DWT 7-level subbands range

Subband	Kandaswamy $F_s = 11025 \text{ Hz}$	Hashemi $F_s = 8000 \text{ Hz}$
A7	0-43.07	0-31.25
D7	43.07-86.13	31.25-62.5
D6	86.13-172.26	62.5-125
D5	172.26-344.53	125-250
D4	344.53-689.06	250-500
D3	689.06-1378.13	500-1000
D2	1378.13-2756.25	1000-2000
D1	2756.25-5512.50	2000-4000

From table.1 [11], the calculation to the parameter of crackle sound will be done in specific subband because crackle only can be heard on 100 until 2kHz[1]. There are 6 features that will be extracted in statisticaly from wavelet coefficient:

- Mean of each subband (μ_{di})

$$\text{mean}(\mu) = \frac{1}{N} \sum_{n=1}^N x_n \quad (3)$$

- Mean of absolute value of each subband(MAV_{di})

$$MAV = \frac{1}{N} \sum_{n=1}^N |x_n| \quad (4)$$

- Variance of each subband(var_{di})

$$var = \frac{1}{N-1} \sum_{n=1}^N (x_n - \mu)^2 \quad (5)$$

- Standard deviation each subband(σ_{di})

$$\text{std}(\sigma) = \sqrt{\frac{1}{N-1} \sum_{n=1}^N (x_n - \mu)^2} \quad (6)$$

- Mean Absolute Deviation each subband(MAD_{di})

$$MAD = \frac{1}{N-1} \sum_{n=1}^N |x_n - ORT| \quad (7)$$

- Zero Crossings of each subband(ZC_{di})

$$MAD = \sum_{n=1}^{N-1} [sgn(x_n * x_{n+1}) \cap |x_n - x_{n+1}| \geq \text{htreshold}] \quad (8)$$

$$sgn(x) = \begin{cases} 1, & \text{if } x \geq \text{threshold} \\ 0, & \text{otherwise} \end{cases}$$

- Average power each subband(p_{di})

$$P = \frac{1}{N} \sum_{n=1}^N (x_n)^2 \quad (9)$$

Mean and variance is the most common and easy implemented feature of the time domain. Mean finds the mean of signal amplitude values over sample length of the signal[13], average power represent frequency distribution from the signal, standard deviation represent the number of frequency distribution change[12]. Zero crossings rate represent the sign-changes along a signal[13]. The average of the absolute deviations represent the average of the absolute deviations of data points from their mean[13].

Restricted Boltzmann Machine(RBM)

Restricted Boltzmann Machine is a stochastic neural network and has 2 layers, namely visible layer and hidden layer. Each unit of the visible layer has an indirect connection to each unit of the hidden layer, with weights associated with it[6]. Each unit of the visible layer and hidden layer are also connected to each bias of the two layers.

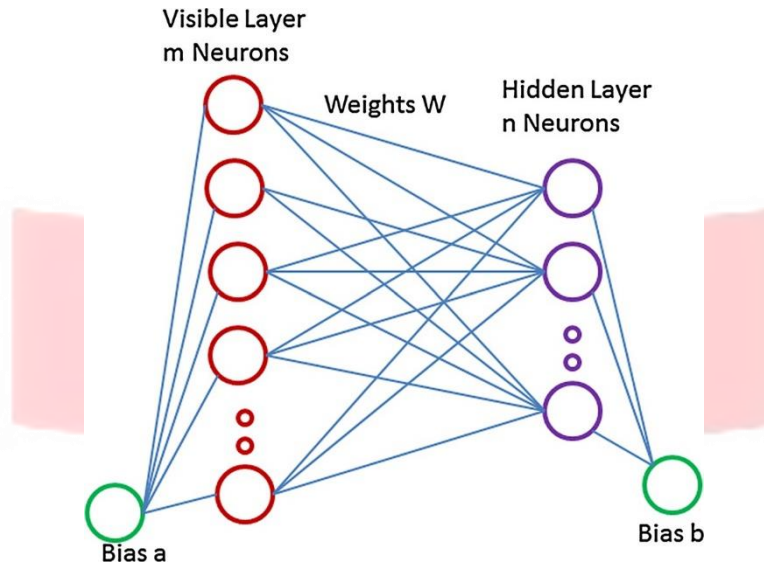


Figure 3 RBM structure[14]

In RBM, each unit in the layer has no connection to the layer itself, in other words the unit in the visible layer does not have a connection with the other units in the layer as well as the hidden layer. The state of the neuron units in the hidden layer will be stochastically updated based on the state of the visible layer and applies to the opposite. In general, probability distributions in hidden layers and visible layers are defined in the energy function. The energy function measures the quality of a joint assignment:

$$E(\mathbf{v}, \mathbf{h}) = -a^T \mathbf{v} - b^T \mathbf{h} - \mathbf{h}^T \mathbf{W} \mathbf{v} \quad (10)$$

where a and b are bias of the visible units and hidden units, respectively. The parameter W is weights of the connection between visible and hidden layer units.

The conditional probability of activation of hidden layer given the visible state $\mathbf{v}$ is computed as:

$$P(h = 1 | \mathbf{v}) = \sigma(\mathbf{W}^T \mathbf{v} + b) \quad (11)$$

Similarly, the conditional probability of activation of visible layer given the hidden state $\mathbf{h}$ is computed as:

$$P(v = 1 | \mathbf{h}) = \sigma(\mathbf{W}^T \mathbf{h} + a) \quad (12)$$

With σ as logistic function as :

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$

In training RBM, Contrastive Divergence or CD are used[14]. The contrastive divergence training is performed with the stochastic steepest ascent. The change of the parameter W by the CD training is given by:

$$\Delta W_{ij} = \epsilon((v_i h_j)_{data} - (v_i h_j)_{recon}) \quad (12)$$

where parameter ϵ is learning rate and v_i is state of visible layer unit given by Eq.(12) and h_j is state of hidden unit given by Eq.(11). The weight matrix W is initialized by some random values and then updated by the value ΔW for each training data set[14]. Similarly, the increments in bias are computed and the bias vectors a and b are updated. The term $(v_i h_j)$ represents the average of the state values products. The subscript “data” is for the value of hidden state computed by Eq. (12), and subscript “recon” is for the

value of visible state computed by Eq. (11). The number of neurons in the visible layers is always equal to input training vector of size m , but the number of neurons in hidden layer n is selected based on the factor by which dimension of training data needs to be reduced [14]. The training data matrix of size $m \times v$ is reduced to feature matrix n linearly independent basis vectors and each represents a W of size $m \times n$, where $n \ll v$. The weight matrix W has unique feature learned from the data [14].

Softmax Classifier

Softmax is a general form of logistic regression that can be used in multi-class classification problems where each class is mutually exclusive [14]. In softmax, the softmax function will replace the sigmoid logistic function in the logistic regression model. The softmax function is stated as follows:

$$P(y = j | z^{(i)}) = \phi_{softmax}(z^{(i)}) = \frac{e^{z^{(i)}}}{\sum_{k=0}^K e^{z_k^{(i)}}} \quad (13)$$

Where net input z is defined as:

$$z = w_1 x_1 + \dots + w_m x_m + b = \sum_{l=1}^m w_l x_l + b = w^T x + b. \quad (14)$$

where w is the vector weight, x is 1 feature vector from the training sample, and b is the unit bias [15].

This softmax function calculates the probability of the $x^{(i)}$ sample training owned by class j with the net weight and input $z^{(i)}$.

Cross Validation

Cross validation (CV) is used to determine if the RBM model used works well. Standards technical used to divide data into training data and testing data. N-fold CV is used to solve problems [11]. In an N-fold CV, data is divided into N data sets, with one data being used as testing data while $N-1$ is used in training data. This process is carried out N times with the final accuracy being the average accuracy of the N measurements [11]. The parameter used for performance assessment is the accuracy described as below:

$$accuracy(\%) = \frac{\text{Number of correctly classified data}}{\text{Total data}} \quad (15)$$

4. Result and Discussion

After doing couple experiment using the system, there were couple result that being acquired using several method, first one is by stacking extracted feature data of each subbands base on its features, and second one is to separate each extracted features subbands. With stack extracted features dataset, it gain 7 features data including class and for separate extracted feature dataset gain is 27 features including class. Whether its stacked dataset or separated dataset, the system will use the same parameter that find out by using hyperparameter tuning with the parameter that consist of learning rate with range [0.1,0.01], number of iterations/sweeps over the training dataset to perform during training with range [10,50,100], number of hidden units with range [100,200,300], and logistic C value with range [1,10,100,1000]. Result of hyperparameter tuning is:

Table 2. best parameter that will be use

parameter	value
logistic_C	1000
rbm_learning_rate	0.01
rbm_n_components	300
rbm_n_iter	10

On logistic, By using bigger C values, the model can increase it's complexity and adjust better to the data but also and therefore, overfit the data. By using this parameter, the result of all method of experiment will be listed below:

The result for stack

Table 3. result of stacked dataset

class	precision	recall	F1- score
normal	0.52	0.99	0.69
crackle	0.28	0.01	0.02
wheeze	0.00	0.00	0.00
crackle+wheeze	0.00	0.00	0.00
Macro average	0.20	0.25	0.18
Weighted average	0.35	0.52	0.36
accuracy	0.52		

From table 2. From normal class, the system able to classify 0.52 of actual normal data out of entire data that predicted being normal and having 0.99 percentage for data that being predicted to normal compared to overall data who actually label normal. percentage of crackle is 0.28 with recall 0.01 meaning that the system able to classify actual crackle labeled data over all data that predicted to be labeled crackle, but still didn't manage to classify predicted labeled crackle data to overall data that actually label crackle. Label wheeze and label crackle+wheeze wasn't been able to be classify by the system. F score is weighted harmonic mean of the precision and recall, where an F-beta score reaches its best value at 1.00 and worst score at 0.00. if 1.00 that mean precision and recall are equally important. From those score it can be say that the system has confuse with stacked data because lot of recall is calculated 0.00 and for accuracy CV using K=10 is 0.52. the data consist of 1237 normal, 646 crackle, 314 wheeze, 172 crackle and wheeze. For result of separated dataset:

Table 4. separated dataset

class	precision	recall	F1- score
normal	0.59	0.92	0.72
crackle	0.39	0.21	0.28
wheeze	0.35	0.03	0.06
crackle+wheeze	0.65	0.12	0.21
Macro average	0.50	0.32	0.32
Weighted average	0.51	0.56	0.48
accuracy	0.56		

In table 3, there are no more 0.00 that mean system able to classify better rather than stacked dataset. Even if recall of wheeze is still low, there's some data that being able to be classify as wheeze. The accuracy CV using K=10 is 0.54 with data consist of 1006 normal, 516 crackle, 235 wheeze, 138 crackle and wheeze.

These two experiment were done with 4 class subject. Because the project call classification of crackle sound then next experiment will be only using crackle and normal only with using same setting as stacked dataset and separated dataset.

Table 5. separated dataset(left) and stacked dataset(right)

class	precision	recall	F1- score
Normal	0.70	0.90	0.79
Crackle	0.60	0.28	0.38
Macro average	0.65	0.59	0.58
Weighted average	0.67	0.69	0.65
accuracy	0.69		

class	precision	recall	F1- score
Normal	0.65	1.00	0.79
Crackle	1.00	0.00	0.00
Macro average	0.83	0.50	0.40
Weighted average	0.77	0.65	0.52
accuracy	0.66		

CV with K=10 left table is 0.69 and right is 0.66. on separated dataset the precision of both class seems high enough which means the system able to calculate percentage on how many data actual label normal or crackle compare to predicted normal or crackle same goes to the stacked dataset even its not high as separated dataset. Recall on normal class from both side is pretty high above 0.90 but for crackle it seem the stacked dataset is got to 0.00 which is lower than separated dataset. Data consist 988 normal and 526 crackle for separated dataset and for stacked dataset consist of 1238 normal and 655 crackle. The accuracy of both is better rather than all 4 class classified result. That happen because theres's the inbalance between the total data with normal, crackle, wheeze, or both class..the system need more data or evenly balance amount of data from each class. This experiment show that even with minimal set of data RBM as its neural network and using softmax to classify able to give a mid high of accuracy. This system still can be improve and by using right choice of features extraction this system will be able achieve a higher accuracy and also the more data used for each class the higher and accurate the system can be.

5. Conclusion

the conclusion base on this experiment for making classification system that able to classify crackle sound is that the system is able to classify but still need more data that the amount of it is balance. There are a lot of thing can be improve from this project from features extraction or adding more hidden layer to it so it not RBM anymore it became DBN or Deep Belief Network, or changing its classifier. This is just a small experiment to find if RBM is able to use for these kind of classification where. With using DWT as feature extraction method is also eases the process of experiment with a bit of improvement it can give extracted features that able to be use by its classifier.

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