# List of Abbreviation

Term	Description	Page of first appearance
DoA	Direction of Arrival Estimation	iii
MVDR	Minimum Variance Distortionless Response	iii
ULA	Uniform Linear Array	iii
SNR	Signal To Noise Ratio	Iii
CW	Continuous Wave	1
LFM	Linear Frequency Modulation	7
NMLF	Non-Linear Frequency Modulation	9

### **Chapter 1 INTRODUCTION**

### 1.1 Background

In 3D radar systems, antenna arrays are used to process beamforming to define the elevation degree, which aims to determine height of target. On the conventional transmitter radar, the cosecant beam pattern is produced by parabolic antenna. In 3D radar, the antennas is an arrays antenna, so to get cosecant beam we must use phase shifter.

To determine the height of a target, the receiver uses Direction of Arrival (DoA) technique. DoA is processed in software. With the DoA process, the target height can be estimated. There are three types of classical DoA algorithm, there are: Minimum Variance Distortionless Response (MVDR) [1], Multiple Signal Classification (MUSIC) [1], and Estimation of Signal Parameters Via Rotational Invariant Techniques (ESPRIT) [1].

MUSIC algorithm is one of the breakthrough algorithms in the field of DoA with eigen analysis techniques on covariance matrix. This algorithm is often called as an eigen based algorithm. The success of this algorithm in detecting several sources at once with very high resolution be the main attraction of this algorithm. Some problems with the MUSIC algorithm are the number of samples for calculating the covariance matrix must be large enough to obtain a statistic stable, and high calculation complexity [1].

ESPRIT algorithm exploits the symmetry structure of the ULA arrangement with the exploitation of this symmetry structure, then the ESPRIT algorithm doesn't do corner-by-corner scanning. However, analytically obtained DoA estimates from received and exploited signals from antenna symmetry structure. The weakness of the ESPRIT algorithm is ability the detection is only half the signal compared to the algorithm MUSIC. The computational process in ESPRIT is quite heavy on the other hand requires high computing resources. Some of these reasons lead to algorithms ESPRIT is less popular in the field of implementation [1].

The MVDR algorithm was proposed by Capon in 1969. The MVDR algorithm was processed by correlating the received signal. The advantage of MVDR algorithm is that this algorithm is robust against noise, and has relatively low complexity. While the drawback of this algorithm is that it has a low detection resolution. Low detection resolution means that it usually cannot distinguished two objects with a small angle of arrival difference. In the term of implementation, MVDR is favourable to MUSIC and ESPRIT. For examples in continuous wave (CW) radar system used MVDR to detect the elevation angle. Simplicity and robustness are the main reason that MVDR is used in these researches.

In this thesis we proposed the modified of MVDR algorithm, with adding matched filter and phase detector before MVDR processing.

## **1.2 Problem Identification**

MVDR algorithm is usually applied to continuous signals, where continuous signals radiate all the time with a fixed frequency value. The MVDR algorithm applied to a continuous signal, the higher the SNR, the better the result.

While the signal emitted on the radar, is a pulse compression signal, where the signal is only emitted at a certain time and has a frequency that changes linearly. Based on the simulation, MVDR is applied to pulse compression signals, resulting in mismatched output.

# 1.3 Objective

The objective of this research is to modify the conventional MVDR so that it can work on chirp signal. The capability of the new algorithm is expected to improve the conventional algorithm in the case of very high noise environment. The modification will include the pre-processing scheme that consist of matched filtering and phase detecting.

## **1.4 Proposed Solution**

The MVDR algorithm will be modified by adding a process between the input signal and the MVDR algorithm process. Before the MVDR algorithm, in the future a matched filter and phase detector will be added.

# **1.5 Expected Results**

With the addition of a process in front of the MVDR algorithm, it is expected that the results of the MVDR algorithm process applied to the pulse compression signal will produce the same results as the MVDR applied to the continuous signal.

### **Chapter 2 BASIC THEORIES**

The antenna used as a basic structure for DoA estimation is an array antenna arranged in a Uniform Linear Array (ULA), and assumed to be an isotropic antenna. The advantage of ULA is that it is simple and uniform. With this uniform and simple structure, the estimation of DoA can be reduced to a simple mathematical equation.

Antennas are arranged vertically against the horizon plane with M elements, and the distance between elements is constant value of d, as shown below:

Figure 2.1 ULA arrangement with antenna distance d

If the signal source comes from a relatively far distance with the angle of arrival  $\theta$ , then the beam reaching each antenna is described as follows Figure 2.2 Far distance Signal are coming to the antenna:



Figure 2.2 Far distance Signal are coming to the antenna

By using the firts antenna as a reference, the direction of the incoming signal will cause a difference in the distance of each element of the adjacent antenna by  $\Delta$  which is expressed as the following equation:

$$\Delta = d\sin(\theta) \tag{2.1}$$

*d* is the distance between the antennas and the angle of arrival  $\theta$ . Assuming the received signal is a sinusoidal signal, the wavelength is  $\lambda$ , and the phase difference between the two antenna elements  $\delta$ :

$$\delta = \frac{2\pi}{\lambda} \, d\, \sin(\theta) \tag{2.2}$$

It is assumed the object reflects a single frequency sinusoidal signal,  $\alpha$  is the send signal's amplitude, f is the signal frequency, and  $\phi$  is a phase signal, so s(t) originating from the object expressed with:

$$s(t) = \alpha \sin\left(2\pi f t + \phi\right) \tag{2.3}$$

If a compound sinusoidal signal is used with a narrow bandwidth, then the signal emitted by the object can be seen as a superposition of  $N_0$  sinusoidal signal with the amplitude  $\alpha_i$ , frequency  $f_i$ , dan phasa  $\phi_i$ , we can determine:

$$s(t) = \sum_{i=1}^{N_0} \alpha_i \sin(2\pi f_i t + \phi_i)$$
(2.4)

The use of sinusoidal signals with a single frequency will provide an advantage for DoA estimation because the phase difference between the two antenna elements can be analyzed easily.



Figure 2.3 Single frequency Sinusoidal signal

The signal received by the receiver antenna is the signal sent by the transmitter which is subject to a delay  $t_p$  and propagation noise. Assuming the single frequency signal used and propagation is Line off Sight (LOS),  $\gamma$  states the attenuation is due to the propagation of the function of time,  $\hat{\alpha}$  is the amplitude of the received signal as a multiplication

between the amplitude of the send signal  $\alpha$ , dan the accept phase is  $\phi_T$  then the signal that reaches the reference antenna (first antenna) is stated as follows:

$$x_R(t) = \gamma s(t - t_p) \tag{2.5}$$

$$= \gamma \alpha \sin(2\pi f(t - t_p) + \phi)$$
(2.6)

$$= \hat{\alpha} \sin(2\pi f t + \phi_T) \tag{2.7}$$

$$x_R(t) = \gamma(t)s(t)e^{-jt_p}$$
(2.8)

Based on Figure (2), with the different phase of  $\delta$  between two adjacent elements and using the first antenna as a reference, then the signal received by the 1st antenna is expressed as follows:

$$x_i(t) = x_R(t - (i - 1)\delta)$$
(2.9)

The signal from the object is expressed with s(t), and  $t_p$  states the propagation delay time, and  $\delta$  is the phase difference between two adjacent antennas, then:

$$x_i(t) = x_R(t)e^{-j(i-1)\delta}$$
(2.10)

$$=\gamma(t)s(t)e^{jt_p}e^{-j(i-1)\delta}$$
(2.11)

By using vectors, the signal received by the x array antenna can be written as follows by:

$$x = a x_R \tag{2.12}$$

Where x is the receive signal matrix, then:

$$x = [x_1(t); x_2(t); \dots; x_M(t)]$$
(2.13)

And vector a is stated with:

$$a = [a \ e^{-j\delta} \ \dots \ e^{-j(M-1)\delta}]^T$$
(2.14)

For each direction of arrival  $(\theta)$ , there will be a vector a, then for each arrival, there will be a vector value. Noise from the environment (n) will add to the receiver signal, so the following equation is obtained:

$$x_n = as + n \tag{2.15}$$

Because the phase difference between two adjacent antennas  $\delta$  is a function of the angle of arrival  $\theta$ , the estimated direction of arrival is to process information on the signal received by the antenna array so that the angle of arrival can be estimated.

For signals coming with a source with an angle of arrival  $\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_s]$ , so:

$$x = A_s s + n \tag{2.16}$$

### 2.1 Classic DoA Estimation Algorithm

The classic DoA algorithm using array antennas has evolved from the 1960s to the 1990s. Some classic milestone algorithms from the DoA estimation technique are using antenna arrays. These algorithms include Delay-and-Sum algorithm or DoS algorithm (Applebaum, 1976), Minimum Variance Distortion less Response algorithm or MVDR algorithm (Capon, 1969), and Multiple Signal Classification algorithm or MUSIC algorithm (Schmidt, 1986). Besides these three algorithms, there is another classic algorithm which is also popular, namely the Estimation of Signal Parameters algorithm via Rotational Invariance Techniques or ESPRIT Algorithms (Roy et al., 1986) [1]. The ESPRIT algorithm exploits the symmetry structure of the ULA arrangement by exploiting this symmetry structure, so the ESPRIT algorithm does not scan corner by angle, but analytically obtains DoA estimates from the received signal and exploits from the antenna symmetry structure. The weakness of the ESPRIT algorithm is that the detection ability of the signal number is only half compared to the MUSIC algorithm. The computational process in ESPRIT is quite heavy on the other hand requires high computing resources. Some of these reasons cause the ESPRIT algorithm to be less popular in the field of implementation. For the purposes of studying the classical algorithm in this section, two classical algorithms will be presented as a comparison, namely the MVDR and MUSIC algorithms. These two algorithms are widely applied in estimating the direction of signal arrival. There is also the ESPRIT algorithm, although it has an advantage in computing time because it does not scan for every angle, but ESPRIT also has problems with large data, because this algorithm requires the calculation of the singular value decomposition of the received signal. The dimension of the received signal must be large enough so that the signal statistics are stable (Lavate et al., 2010).

#### **2.2 MVDR**

This algorithm was introduced by J. Capon (Capon, 1969) in his study using an array of seismic sensors to determine the source of the earthquake [2]. To improve the stability of signal estimation in noisy environments, this algorithm collects considerable data and processes it into a Rxx covariance matrix. This covariance matrix is calculated by Equation (I.19)

$$R_{xx} = E(xx^{T}) = \frac{1}{N_s} xx^{T}$$
 (2.17)

So, obtained power spectrum MVDR  $(P_{MVDR})$  as follows:

$$P_{MVDR}(\theta) = \frac{1}{a^T(\theta)R_{xx}^{-1}a(\theta)}$$
(2.18)

The value of  $P_{MVDR}(\theta)$  calculated for each value  $\theta$  ranges from -90° to 90° and the addition of sufficient  $d\theta$  is small, for example 1°. Vector a ( $\theta$ ) represents the steering vector in the direction  $\theta$ . By describing the spectrum value of  $P_{MVDR}(\theta)$  as a function  $\theta$ , the direction of signal arrival is estimated at the value of  $\theta$  which gives  $P_{MVDR}(\theta)$  the highest values [2].

In terms of performance, algorithms have good resistance to noise, but do not have high source separation resolution. Research on the influence of interference from external sources on the performance of the MVDR algorithm was investigated by Zoltowski [3]. In that study, Zoltowski showed the MVDR algorithm is sensitive in environments with high interference so that the accuracy of the estimation decreases in that environment. Another problem with the MVDR algorithm is that the number of samples used must be large enough for the covariance matrix to be statistically stable, and the inverse matrix calculation and scanning at all angles causes this algorithm to require large computing resources [2].

MVDR algorithm is considered as a standard classical algorithm for estimating the direction of signal arrival. The performance of this algorithm is often used as a comparison. For example, Goronitsky and Rao use MVDR as a classical comparison algorithm against the proposed sparse reconstruction algorithm.

### **2.3 Pulse Compression**

The maximum detection on a radar is determined by how much the echo signal is received. To get strong echo reflections, a large amount of transmit energy is needed. Meanwhile, when passing through the propagation channel, the echo signal experiences a lot of distortion, both from AWGN or from multipath fading, so that the echo received by the receiver is no longer patterned.

The pulse compression algorithm basically uses the auto correlation method on the receiver. The signal received by the receiver will be correlated with a coefficient whose value is the same as the amplitude of the signal sent by the transmitter. The correlation output will be highest when the echo value correlated has the same pattern as the transmitted one. Thus, the pulse compression algorithm is very resistant to AWGN channels.

There are 2 types of pulse compression algorithms, namely linear frequency modulation (LFM) and Non-linear Frequency Modulation (LFM).

### 2.3.1 Linear Frequency Modulation (LFM)

Signal modulation in frequency and phase can be used for wide bandwidth. LFM is widely used in modern radar systems. In LFM, the frequency can be implemented with the upchirp linear phase, or with the down-ward (down-chirp) phase. With a pulse width  $\tau_0$  and bandwidth B, the LFM can be formulated as follows:

$$\phi(t) = 2\pi \left( f_0 t + \frac{\mu}{2} t^2 \right) \qquad -\frac{\tau_0}{2} \le t \le \frac{\tau_0}{2} \tag{2.19}$$

With f\_0 the center radar frequency, and  $\mu = B / t_0$  is the LFM coefficient, the frequency can be formulated as follows:

$$f(t) = \frac{1}{2\pi} \frac{d}{dt} \phi(t) = f_0 + \mu t \quad -\frac{\tau_0}{2} \le t \le \frac{\tau_0}{2}$$
(2.20)

Same with the above equation, for down-chirp signal can be formulated as follows:

$$\phi(t) = 2\pi \left( f_0 t - \frac{\mu}{2} t^2 \right) \qquad -\frac{\tau_0}{2} \le t \le \frac{\tau_0}{2} \qquad (2.21)$$



Figure 2.4 Up-chirp & Down-chirp phase of LFM



Figure 2.5 Up-Chirp LFM



Figure 2.6 Down-Chirp LFM

The ability of the receiver to increase distance resolution compared to conventional systems is called the pulse compression ratio (PCR). PCR can be expressed as a comparison of the resolution of the distance between unmodulated pulses with length t to modulated pulses of equal length and bandwidth B, can be formulated as follows:

$$PCR = \frac{c \ast \frac{t}{2}}{\frac{c}{2B}} = Bt \tag{2.22}$$

2.3.2 Non-Linear Frequency Modulation (NLFM)

The disadvantage of LFM is the time sidelobe, as shown in the Figure 7.



Figure 2.7 Time Sidelobe chirp LFM

To reduce the time sidelobe can be done by weighting the amplitude but it will reduce Pulse Compression Grating (PCG). Another way is to use Non-Linear Frequency Modulation so that it doesn't reduce the value of PCG. There are 2 types of Non-Linear FM symmetry and non-symmetry waveform, as shown below:



Figure 2.8 Non-linear FM chirp

### 2.3 Matched Filter

Matched filter with pulse duration  $\tau$ :

$$x(t) = \begin{cases} 1, \ 0 \le t \le \tau \\ 0, \ otherwise \end{cases}$$
(2.23)

The corresponding matched filter impulse response:

$$h(t) = ax * (T_M - t)$$
 (2.24)

$$h(t) = \begin{cases} \alpha, \ T_M - \tau \le t \le T_M \\ 0, \ otherwise \end{cases}$$
(2.25)

The working principle of a matched filter is a correlation process. The chirp signal received at the receiver will be processed matched filter by correlating it with a filter that has a transpose coefficient value of the chirp sent by the transmitter. The chirp transmit signals and output matched filter can be observed in Figure 9.



Figure 2.9 Chirp & Matched Filter

# Chapter 3 MVDR MODIFICATION SCHEME WITH INPUTS CHIRP SIGNAL

In the classic MVDR algorithm, the chirp signal is directly processed using MVDR. Chirp generate from the same source will be inputted into the MVDR algorithm and the MVDR value calculated as show as Figure 3.1



Figure 3.1: Classical MVDR

The input used in the conventional MVDR algorithm is the sine signal with frequency and single-phase values. In this thesis the pulse compression LFM (chirp) signal will be used as the input. The proposed scheme is as follows:



Figure 3.2: Modified MVDR System Design